A NEW FORMULATION FOR MODELING BLOOD FLOW IN DEFORMABLE ARTERIES: THE COUPLED MOMENTUM METHOD FOR FLUID-SOLID INTERACTION PROBLEMS

A. Figueroa[a], **K. Jansen**[b], **T. Hughes**[c] and **C. Taylor**[d]

[a]Department of Mechanical Engineering
Stanford University
Durand 224. Stanford, CA 94305-4038
cafa@stanford.edu

[b]Department of Mechanical Engineering
Rensselaer Polytechnic Institute
Troy, NY
kjansen@scorec.rpi.edu

[c]Department of Aerospace
Engineering and Engineering Mechanics
TICAM
Austin, TX
tjr_hughes@hotmail.com

[d]Departments of Mechanical Engineering,
Surgery and Pediatrics
Stanford University
Durand 213. Stanford, CA 94305-4038
taylorca@stanford.edu

Computational techniques applied to solve the equations of blood flow in three-dimensional domains have only examined the velocity field (not pressure) and have generally treated the wall as rigid. Computational methods for fluid-structure interaction (e.g. the ALE formulation [1]) have thus far proven to be computationally too expensive and unstable for clinical applications including surgical planning. A new formulation for modeling blow flow in deformable arteries termed the *Coupled Momentum Method for Fluid-Solid Interaction* (CMM-FSI) is presented. The formulation considers a strong coupling of the fluid and solid mechanics degrees of freedom and a linear elastic membrane formulation for the vessel wall. The coupling of the fluid and the solid is achieved by generalizing the idea developed by Womersley in his derivation of an analytical solution for pulsatile blood flow in an elastic vessel [2]. This approach considers the solid mechanics problem as a special boundary condition for the fluids problem, by relating the unknown traction t^f_i on the lateral wall of the fluid domain due to the vessel wall with the body force of the solids problem b^s_i. Lastly, for computational efficiency, the fluid domain is kept fixed and a linearized kinematics formulation is proposed for the vessel wall. The weak form of the Navier-Stokes equations in advective form ([3,4]) for a deformable domain is

$$\int_{\Omega}\left\{w_i\left(v_{i,t}+v_j v_{i,j}-f_i\right)+w_{i,j}\left(-p\delta_{ij}+\tau_{ij}\right)-q_{,i}v_i\right\}dx+\int_{\Gamma_h}\left\{-w_i h_i+qv_n\right\}ds$$
$$+\int_{\Gamma_s} qv_n ds + h\int_{\Gamma_s}\left\{w_i v_{i,t}+w_{(i,j)}c_{ijkl}u_{(k,l)}\right\}ds - h\int_{\partial\Gamma_h} w_i h_i^s\, dl = 0$$

where v_i and p are the *ith* component of velocity and the pressure; w_i and q the weighting functions for velocity and pressure; τ_{ij} the viscous stress tensor; f_i the body force of the fluid problem; h_i is the prescribed traction on the Neumann fraction of the boundary (Γ_h); u_i is the *ith* component of the displacement of the vessel wall domain (Γ_s), and h^s_i is the traction on the boundary of the vessel wall domain. In this work, the CMM-FSI method will be described, and the finite element formulation and recent results presented.

References

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